OCR
Oxford Cambridge and RSA

Tuesday 21 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

6385797585

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Find the exact value of $\int_0^{\frac{1}{2}\pi}(1+\cos\frac{1}{2}x)\,dx$. **[3]**

2 The functions f(x) and g(x) are defined by $f(x) = \ln x$ and $g(x) = 2 + e^x$, for $x > 0$.

Find the exact value of x, given that $fg(x) = 2x$. **[5]**

3 Find $\int_1^4 x^{-\frac{1}{2}}\ln x\,dx$, giving your answer in an exact form. **[5]**

4 By sketching the graphs of $y = |2x+1|$ and $y = -x$ on the same axes, show that the equation $|2x+1| = -x$ has two roots. Find these roots. **[4]**

5 The volume $V\,m^3$ of a pile of grain of height h metres is modelled by the equation

$$V = 4\sqrt{h^3+1} - 4.$$

(i) Find $\frac{dV}{dh}$ when $h = 2$. **[4]**

At a certain time, the height of the pile is 2 metres, and grain is being added so that the volume is increasing at a rate of $0.4\,m^3$ per minute.

(ii) Find the rate at which the height is increasing at this time. **[3]**

6 Fig. 6 shows part of the curve $\sin 2y = x - 1$. P is the point with coordinates $(1.5, \frac{1}{12}\pi)$ on the curve.

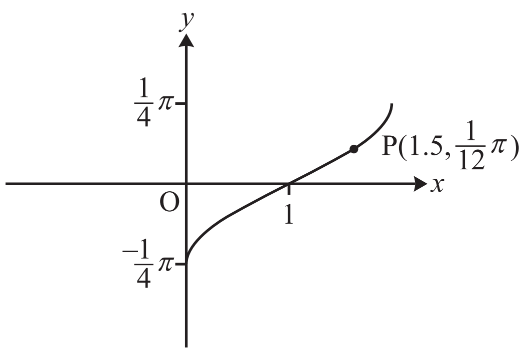


Fig. 6

(i) Find $\frac{dy}{dx}$ in terms of y.

Hence find the exact gradient of the curve $\sin 2y = x - 1$ at the point P. **[4]**

The part of the curve shown is the image of the curve $y = \arcsin x$ under a sequence of two geometrical transformations.

(ii) Find y in terms of x for the curve $\sin 2y = x - 1$.

Hence describe fully the sequence of transformations. **[4]**

7 You are given that n is a positive integer.

By expressing $x^{2n} - 1$ as a product of two factors, prove that $2^{2n} - 1$ is divisible by 3. **[4]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = \frac{x}{\sqrt{x+4}}$ and the line $x = 5$. The curve has an asymptote l.

The tangent to the curve at the origin O crosses the line l at P and the line $x = 5$ at Q.

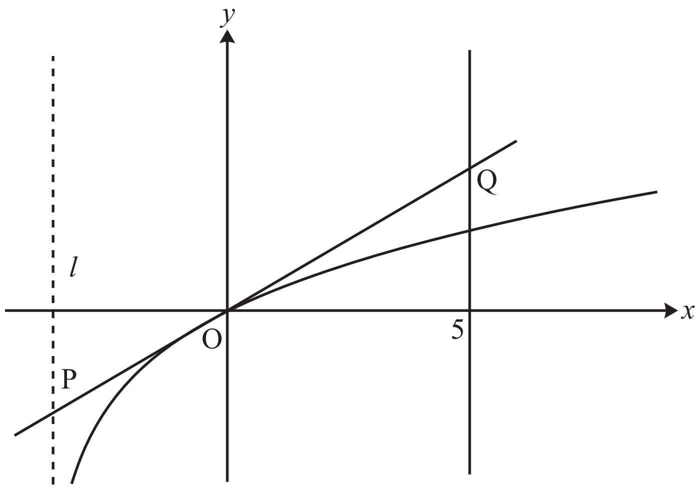


Fig. 8

(i) Show that for this curve $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x+8}{2(x+4)^{\frac{3}{2}}}$. **[5]**

(ii) Find the coordinates of the point P. **[4]**

(iii) Using integration by substitution, find the exact area of the region enclosed by the curve, the tangent OQ and the line $x = 5$. **[9]**

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \mathrm{e}^{2x} + k\,\mathrm{e}^{-2x}$ and k is a constant greater than 1.

The curve crosses the y-axis at P and has a turning point Q.

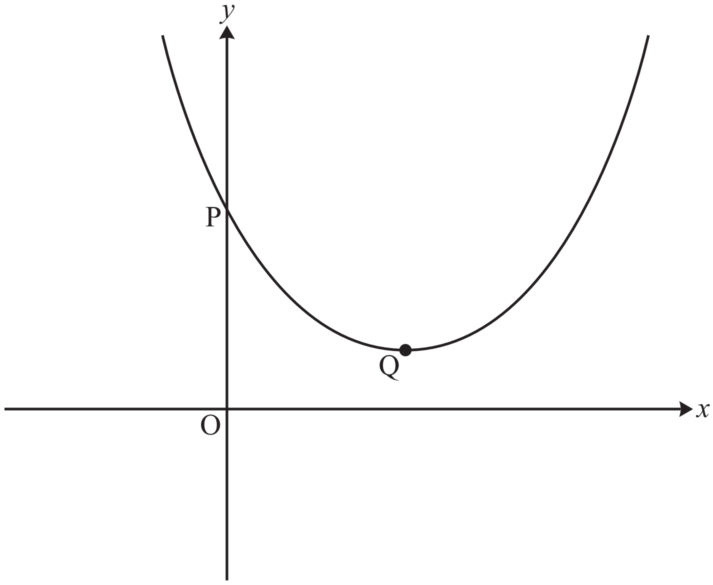


Fig. 9

(i) Find the y-coordinate of P in terms of k. **[1]**

(ii) Show that the x-coordinate of Q is $\frac{1}{4}\ln k$, and find the y-coordinate in its simplest form. **[5]**

(iii) Find, in terms of k, the area of the region enclosed by the curve, the x-axis, the y-axis and the line $x = \frac{1}{2}\ln k$. Give your answer in the form $ak + b$. **[4]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \mathrm{f}(x + \frac{1}{4}\ln k)$.

(iv) (*A*) Show that $\mathrm{g}(x) = \sqrt{k}\,(\mathrm{e}^{2x} + \mathrm{e}^{-2x})$. **[3]**

(*B*) Hence show that $\mathrm{g}(x)$ is an even function. **[2]**

(*C*) Deduce, with reasons, a geometrical property of the curve $y = \mathrm{f}(x)$. **[3]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Tuesday 21 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:
- Question Paper 4753/01 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

 OCR is an exempt Charity

Section A (36 marks)

1

2

3	

4

5 (i)	
5 (ii)	

6 (i)	
6 (ii)	

7

Section B (36 marks)

8 (i)

8 (ii)	

8(iii)

9 (i)	
9 (ii)	

9 (iii)	

9(iv)(*A*)	
9(iv)(*B*)	

9(iv)(*C*)	

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4753**: Methods for Advanced Mathematics

Advanced GCE

Mark Scheme for June 2016

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

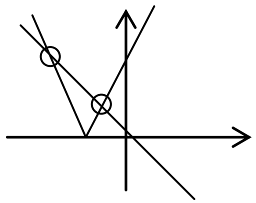
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$\int_0^{\frac{\pi}{2}}(1+\cos\frac{1}{2}x)\,\mathrm{d}x=\left[x+2\sin\frac{1}{2}x\right]_0^{\frac{\pi}{2}}$	B1	$\left[x+2\sin\frac{1}{2}x\right]$	
			$=\frac{\pi}{2}+2\sin\frac{\pi}{4}[-0]$	M1	substituting limits (upper – lower)	allow 1 slip
			$=\frac{\pi}{2}+\sqrt{2}$	A1cao	must be exact, not $2/\sqrt{2}$	isw from correct answer seen
				[3]		
2			fg(x) = ln(2 + e^x)	M1	condone missing brackets	
			$\Rightarrow$ ln(2 + e^x) = 2x			
			$\Rightarrow$ 2 + e^x = e^{2x}	A1		
			$\Rightarrow$ e^{2x} – e^x – 2 [= 0]	M1	Rearranging into a quadratic in e^x	may be implied from both correct roots
			$\Rightarrow$ (e^x – 2)(e^x + 1) = 0, e^x = 2, −1	A1	obtaining roots 2, −1	−1 root may be inferred from factorising
			$\Rightarrow$ e^x = 2, x = ln 2	A1	x = ln 2 only, not from ww	x = ln (−1) is A0
				[5]		
3			let u = ln x, u' = 1/x, v'= $x^{-1/2}$, v = $k\,x^{1/2}$	M1	soi ($k \neq 0$)	
			$\int x^{-1/2}\ln x[\mathrm{d}x]=\left[2x^{1/2}\ln x\right]-\int 2x^{1/2}.\frac{1}{x}[\mathrm{d}x]$	A1		
			$=\left[2x^{1/2}\ln x\right]-\int 2x^{-1/2}[\mathrm{d}x]$	M1	$x^{1/2}/x = x^{-1/2}$ or $1/x^{1/2}$ seen	
			$=\left[2x^{1/2}\ln x-4x^{1/2}\right]_1^4$	A1	$2x^{1/2}\ln x - 4x^{1/2}$	may be integrated separately
			= 4 ln 4 – 8 – (2ln 1 – 4)			
			= 4 ln 4 − 4	A1cao	oe (eg ln 256–4) but must evaluate ln1=0	mark final answer
				[5]		

Question			Answer	Marks	Guidance	
4				M1	Sketch of $y = \|2x + 1\|$	condone no intercept labels, but must be a 'V' shape with vertex on −ve x axis
				A1	$y = -x$ and two intersections indicated	
			$x = -1$ $x = -1/3$	B1 B1 [4]	not from ww, condone (−1, 1) not from ww, condone (−1/3, 1/3)	squaring: $(2x+1)^2 = x^2 \Rightarrow 3x^2 + 4x + 1 = 0$ $\Rightarrow (3x+1)(x+1) = 0$, $x = -1, -1/3$
5	(i)		$dV/dh = 4.½(h^3 + 1)^{-1/2}\ .3h^2$ when $h = 2$, $dV/dh = 8$	M1 A1 M1 A1cao [4]	chain rule correct substituting $h = 2$ into their derivative	their deriv of $4u^{1/2}$ × their deriv of h^3+1
5	(ii)		$dV/dt = 0.4$ $dV/dt = dV/dh \times dh/dt$ $0.4 = 8 \times dh/dt \Rightarrow dh/dt = 0.05$ (m per min)	B1 M1 A1cao [3]	soi o.e. 0.05 or 1/20	condone r for t any correct chain rule in V, h, t (or r)
6	(i)		$2\cos 2y\ dy/dx = 1$ $\Rightarrow dy/dx = 1/(2\cos 2y)$ when $x = 1½$, $y = \pi/12$, $dy/dx = 1/(2\cos(\pi/6))$ $= 1/\sqrt{3}$	M1 A1 M1* A1 [4]	$k \cos 2y\ dy/dx = 1$ substituting $y = \pi/12$ *dep 1st M1 or $\sqrt{3}/3$	or $dx/dy = k \cos 2y$, $k\cos 2y dy = dx$ $dy/dx = k \cos 2y$ is M0 isw from correct exact answer
6	(ii)		$2y = \arcsin(x - 1)$ $\Rightarrow y = ½ \arcsin(x - 1)$ translation of 1 unit in positive x-direction [one-way] stretch s.f. ½ in y-direction	M1 A1 B1 B1 [4]	 or $½ \sin^{-1}(x - 1)$ or translation $\begin{pmatrix}1\\0\end{pmatrix}$ not 'shrink', 'squash' etc	 allow 'shift', but not 'move', vector only is B0 transformations can be in either order

Question			Answer	Marks	Guidance	
7			$x^{2n} - 1 = (x^n - 1)(x^n + 1)$	B1		
			one of 2^n–1, 2^n+1 is divisible by three	M1		award notwithstanding false reasoning
			$2^n - 1$, 2^n and $2^n + 1$ are consecutive integers;			condone 'factor' for 'multiple'
			one must therefore be divisible by 3;	A1		
			but 2^n is not, so one of the other two is	A1	if justified, correct reason must be given	
			or			
			2^n is not div by 3, and so has remainder 1 or 2 when divided by 3; if remainder is 1, $2^n - 1$ is div by 3; if remainder is 2, then $2^n + 1$ is div by 3	A2		
			[so $2^{2n} - 1$ is divisible by 3]			
				[4]		
8	**(i)**		$\dfrac{dy}{dx} = \dfrac{(x+4)^{1/2}.1 - x.\frac{1}{2}(x+4)^{-1/2}}{[(x+4)^{1/2}]^2}$	M1	quotient rule: $v \times$ their $u' - u \times$ their v', and correct denominator	or product rule
				B1	½ $u^{-1/2}$ soi	or −½ $u^{-3/2}$ (PR)
				A1	correct expression	PR: $x(-½)(x+4)^{-3/2} + (x+4)^{-1/2}$
			$= \dfrac{x+4-\frac{1}{2}x}{(x+4)^{3/2}} = \dfrac{\frac{1}{2}x+4}{(x+4)^{3/2}} = \dfrac{x+8}{2(x+4)^{3/2}}$ *	M1	factoring out $(x + 4)^{-1/2}$ o.e.	$= (x+4)^{-3/2}(-½x + x + 4)$
				A1	**NB AG**	
				[5]		
8	**(ii)**		[asymptote is] $x = -4$	B1	soi	but from correct working
			gradient of tangent at O= $8/(2 \times 4^{3/2}) = ½$	B1	gradient = ½	
			eqn of tangent is $y = ½x$	B1	o.e. e.g. using gradient	
			When $x = -4$, $y = -2$, so $(-4, -2)$	B1		
				[4]		

Question			Answer	Marks	Guidance	
8	(iii)		let $u = x + 4$, $du = dx$	B1	or $dx/du = 1$	or $v^2 = x+4$, $2v dv/dx = 1$ or $2v dv = dx$ oe e.g. $dv/dx = ½ (x+4)^{-1/2}$
			$\int_0^5 \frac{x}{(x+4)^{1/2}} dx = \int_4^9 \frac{u-4}{u^{1/2}} du$	B1	$\int \frac{u-4}{u^{1/2}}[du]$	$\int \frac{v^2-4}{v} 2v[dv]$
			$= \int_4^9 (u^{1/2} - 4u^{-1/2}) du$	B1	$u^{1/2} - 4u^{-1/2}$ or $u^{1/2} - 4/u^{1/2}$, or $\sqrt{u} - 4/\sqrt{u}$	$\int (2v^2 - 8)[dv]$
			$= \left[\frac{2}{3}u^{3/2} - 8u^{1/2}\right]_4^9$	B1	$\left[\frac{2}{3}u^{3/2} - 8u^{1/2}\right]$ o.e.	$\left[\frac{2}{3}v^3 - 8v\right]$
			$= (18 - 24) - (16/3 - 16)$	M1	substituting correct limits (upper – lower)	0, 5 for x; 4,9 for u; 2,3 for v
			$= 14/3$	A1cao		
			or (following first 2 marks)			by parts with no substitution:
			let $v = u - 4, w' = u^{-1/2}, v' = 1, w = 2u^{1/2}$	M1		$u = x, u' = 1, v' = (x+4)^{-1/2}, v = 2(x+4)^{1/2}$ M1
			$\int_4^9 (u-4)u^{-1/2} du = \left[2u^{1/2}(u-4)\right]_4^9 - \int_4^9 2u^{1/2} du$	A1		$=[2x(x+4)^{1/2}] - \int 2(x+4)^{1/2}$ A1 $= \left[2x(x+4)^{1/2} - \frac{4}{3}(x+4)^{3/2}\right]_0^5$ A1
			$= \left[2u^{1/2}(u-4) - \frac{4}{3}u^{3/2}\right]_4^9$	A1		$=14/3$ A1 (so max of 4/6)
			$= 14/3$	A1cao		
			y- coordinate of Q is 2½	B1	(soi)	or $\int_0^5 \frac{1}{2}x\,dx$ M1
			Area of triangle = ½ × 5 × 5/2 = 25/4	B1		$= \left[\frac{1}{4}x^2\right]_0^5 = 25/4$ A1
			Enclosed area = 25/4 – 14/3 = $1\frac{7}{12}$	B1	or 19/12, or $1.58\dot{3}$	isw from correct exact answer
				[9]		

Question			Answer	Marks	Guidance	
9	(i)		$1 + k$	B1 [1]		
9	(ii)		$f'(x) = 2\,e^{2x} - 2\,k\,e^{-2x}$ $f'(x) = 0 \Rightarrow 2\,e^{2x} - 2\,k\,e^{-2x} = 0$ $\Rightarrow e^{2x} = k\,e^{-2x}$ $\Rightarrow e^{4x} = k$, $4x = \ln k$, $x = ¼ \ln k$ * $y = e^{(½ \ln k)} + k\,e^{(-½ \ln k)}$ $= \sqrt{k} + k/\sqrt{k} = 2\sqrt{k}$	B1 M1 A1 M1 A1cao [5]	 their derivative = 0 **NB AG** substituting $x = ¼ \ln k$ into f(x) or $2k^{1/2}$	
9	(iii)		Area = $\int_0^{\frac{1}{2}\ln k}(e^{2x} + ke^{-2x})[\mathrm{d}\,x]$ $= \int_0^{\frac{1}{2}\ln k}(e^{2x} + ke^{-2x})\,\mathrm{d}\,x = \left[\frac{1}{2}e^{2x} - \frac{1}{2}ke^{-2x}\right]_0^{\frac{1}{2}\ln k}$ $= ½\,k - ½ - ½ + ½\,k$ $= k - 1$	B1 B1 M1 A1 [4]	correct integral and limits (soi) $\left[\frac{1}{2}e^{2x} - \frac{1}{2}ke^{-2x}\right]$ $e^{\ln k} = k$ or $e^{-\ln k} = 1/k$ (soi)	
9	(iv)	(*A*)	$g(x) = e^{2(x + ¼ \ln k)} + ke^{-2(x + ¼ \ln k)}$ $= e^{2x}.e^{½ \ln k} + k\,e^{-2x}.\,e^{-½ \ln k}$ $= (e^{\ln k})^{1/2}\,e^{2x} + k.(e^{\ln k})^{-1/2}\;\;e^{-2x}$ $= k^{1/2}\,e^{2x} + k.k^{-1/2}.\,e^{-2x}$ $= \sqrt{k}\,(e^{2x} + e^{-2x})$ *	M1 M1 A1 [3]	Substitute $x + ¼ \ln k$ for x in f(x) $e^{p+q} = e^p \times e^q$ used **NB AG** – must show enough working	condone missing brackets e.g. $k^{1/2}\,e^{2x} + k.k^{-1/2}.\,e^{-2x}$
9	(iv)	(*B*)	$g(-x) = \sqrt{k}\,(e^{-2x} + e^{2x})$ $= g(x)$ so g is even	M1 A1 [2]	substituting $-x$ for x must include $g(-x) = g(x)$, and either define an even function or conclude that g is even	condone 'f' used instead of 'g' for M1 not $f(-x) = f(x)$ for A1
9	(iv)	(*C*)	g(x) is symmetrical about the y-axis, and f(x) is g(x) translated ¼ ln k in x-direction so f(x) is symmetrical about $x = ¼ \ln k$	B1 B1 B1 [3]	 allow 'shift' or 'move' allow final B1 even if unsupported	 or g is f translated – ¼ ln k or incorrectly supported

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

4753 Methods for Advanced Mathematics (C3 Written Examination)

General Comments:

There was a very pleasing standard of work produced on this paper. The majority of candidates were clearly well prepared, and there were many excellent scripts, with a fifth of the candidates scoring over 65 marks, and 90% scoring over 30 marks. There appeared to be an improvement in performance on some topics, such as the modulus function, implicit differentiation and inverse trigonometric functions. There was little evidence of learners running out of time. Standards of presentation were as variable as ever, but many scripts were well presented and clearly argued.

Candidates sometimes offer repeated attempts at questions. Under these circumstances, learners should be told to cross out the ones which they do not wish to be marked. Otherwise, we mark the final complete attempt, notwithstanding if it scores fewer marks than previous ones!

Comments on Individual Questions:

Section A

1. This proved to be a straightforward starter question, with 80% of candidates scoring full marks. Some candidates stopped at $\pi/2+2\sin\pi/4$, presumably because they did not appreciate that 'value of' means numerical. A few weaker candidates confused differentiation and integration, either giving the wrong coefficient or sign for the $\sin x/2$ term.

2. Virtually all candidates formed the composite function in the correct order to obtain $fg(x) = \ln(2+e^x)$. A few then simplified this to $\ln 2 + x$ and therefore made no further progress. Of those who did correctly proceed to $2 + e^x = e^{2x}$, a substantial minority then incorrectly took logs of each side to reach $\ln 2 + x = 2x$. Of those who correctly rearranged the equation into a quadratic in e^x, nearly all then gained full marks, correctly rejecting the $e^x = -1$ solution.

3. Integration by parts was well understood, with just under half candidates scoring full marks for this question. Very occasionally, candidates took $u = x^{-1/2}$ and $v' = \ln x$, and were unable to score any marks. With u and v correct, the next hurdle is to simplify the $2x^{1/2}.1/x$ integrand, and some failed at this stage, and attempted to integrate the product term by term. Having negotiated this successfully, most got full marks, though very occasionally the final answer was spoiled by using $4\ln 4 = \ln 16$.

4. Sketches of the modulus function with $y = -x$ were generally well done, though quite a few lost a mark for neither clearly indicating the intercepts nor making a clear statement that there were two of them. The roots were then usually found correctly, with less evidence of faulty modulus algebra than in recent years.

5. This question was extremely well answered, with the majority of candidates scoring full marks.

5(i). The chain rule on V was successfully negotiated by over half the candidates, and then correctly evaluated at $x = 2$.

5(ii). Virtually everyone who scored 4 for part (i) went on to apply the chain rule $dV/dt = dV/dh \times dh/dt$, or some variation of it, to get full marks here. The rest usually earned the first two of the three marks.

6. This question as also very well done, with half the candidates scoring full marks.

6(i). The implicit differentiation was well understood, though there were the usual blemishes from mixing up the derivative and integral formulae for sin 2*y*. A few candidates re-arranged the equation to get *x* in terms of *y*, then found d*x*/d*y*, and then the reciprocal d*y*/d*x*.

6(ii). Re-arranging the given implicit equation to give $y = \frac{1}{2}\arcsin(x - 1)$ was well understood, and the transformations were usually accurately described. Note that the preferred terms here are 'translation' and 'one-way stretch'.

7. The first B1 for factorising $x^{2n} - 1$ was well done, but convincing proofs of the divisibility of $2^{2n} - 1$ by 3 were few and far between. We awarded M1 if candidates recognised that either $2^n–1$ or 2^n+1 were divisible by 3, and two 'A' marks for proving this. The next 'A' mark was gained for stating that the consecutive numbers $2^n−1$, 2^n and 2^{n+1} must include a multiple of 3, and the final mark for stating that 2^n is **not** divisible by 3; however, many candidates wrongly stated that 2^n was even and therefore not divisible by 3, or that two consecutive odd numbers must include a multiple of 3. The most elegant alternative solution seen was:

$x^{2n} - 1 = (x^2 - 1)(x^{2n-2} + x^{2n-4} + \ldots + 1) \Rightarrow 2^{2n} - 1 = (2^2-1)(2^{2n-2} + 2^{2n-4} + \ldots + 1) = 3m$, where m is an integer.

The language used by candidates in their explanations was often rather imprecise. In particular, the terms 'factor' and 'multiple' were often used incorrectly.

Section B

8. Most candidates scored well on this question, which covered calculus topics such as the product or quotient rule for differentiation and integration by substitution, which are generally well understood by learners.

8(i). The first three marks here were usually earned, though a minority of weaker candidates mixed up the product and quotient rules, for example using $v = (x+4)^{-1/2}$ in their quotient rule. The factorisation required to achieve the given result was less successfully done, but just over half the candidates still managed full marks here. There were a lot of repeated attempts at this, for example using the product rule when they got stuck with manipulating their quotient rule expression.

8(ii). This proved to be a straightforward 4 marks earned by over 70% of scripts. The asymptote and the gradient and equation of the tangent at the origin were usually correctly found, followed by the coordinates of Q.

8(iii). This 9-mark question required careful extended work from candidates, but there was a pleasing response, with just under half the scripts earning full marks. The first six of these were for finding the area under the function using substitution. Here, as usual, notation sometimes left something to be desired, with missing d*u*'s or d*x*'s, integral signs, inconsistent limits, etc. Most of this we condoned, but we did require $du/dx = 1$ or its equivalent to be stated. The final three marks depended upon the correct coordinates for the point Q being found in part (ii). Occasionally the triangle area was found using $\int \frac{1}{2} x \, dx$.

9. The calculus here was not particularly demanding, requiring only the derivative and integral of e^{kx}; but the simplification of expressions using the laws of logarithms and exponentials proved to be quite testing and found out quite a few candidates.

9(i). This was an easy write-down for virtually all candidates, except those few who did not know that $e^0 = 1$.

9(ii). The first two marks were pretty universally earned, but deriving $x = \frac{1}{4}\ln k$, together with the final 'A' mark for getting $2\sqrt{k}$, caused a few problems, with some inaccurate logarithm work. For example, $e^{1/2 \ln k} = \frac{1}{2}k$ was a commonly seen misconception.

9(iii). The integration was usually correct, but, thereafter, as in part (ii), the simplification to arrive at $k - 1$ proved to be tricky, with similar errors being made.

9(iv)(*A*). Most attempts correctly substituted $x + \frac{1}{4} \ln k$ for x in f(x) to gain the first M mark, but we needed to see clear evidence of how this simplifies to the given result. Often candidates seemed to be working backwards from this without really understanding the process.

9(iv)(*B*). The definition of an even function was well known, but sometimes the structuring of the proof was indecisively presented. Some used 'f' instead of 'g' (here, f is indeed **not** an even function!), and we required to see either a clear statement of the definition of an even function, or a clear conclusion that g is therefore even. The structure '$g(-x) = \ldots = \ldots = g(x) \Rightarrow$ g is even' is the most transparent formulation to use in such proofs, rather than starting them by stating that $g(-x) = g(x)$, viz the result they are trying to prove!

9(iv)(*C*). The argument here proved beyond most candidates, with only 20% getting full marks. Many stated that f was an even function, perhaps thinking that any line of symmetry sufficed. Sometimes it was indeed a little difficult to decide whether candidates were referring to f or g in their answers.

Oxford Cambridge and RSA

GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	57	52	47	42	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	49	42	35	29	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	64	57	51	45	39	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	59	53	48	43	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	60	54	48	43	38	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	60	54	49	44	39	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	67	61	55	49	43	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	59	53	47	41	36	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	60	53	46	40	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	55	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	60	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	60	54	48	42	37	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	48	43	38	34	30	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	55	50	45	40	36	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	55	49	44	39	33	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0

			UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	59	52	46	40	34	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	59	52	46	40	34	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	48	43	38	34	30	0
		UMS	100	80	70	60	50	40	0

Level 3 Certificate and FSMQ raw mark grade boundaries June 2016 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

Level 3 Certificate Mathematics for Engineering

			Max Mark	a*	a	b	c	d	e	u
H860	01 Mathematics for Engineering		This unit has no entries in June 2016							
H860	02 Mathematics for Engineering									

Level 3 Certificate Mathematical Techniques and Applications for Engineers

			Max Mark	a*	a	b	c	d	e	u
H865	01 Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)

			Max Mark	a	b	c	d	e	u
H866	01 Introduction to quantitative reasoning	Raw	72	55	47	39	31	23	0
H866	02 Critical maths	Raw	60	47	41	35	29	23	0
		Overall	132	111	96	81	66	51	0

Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)

			Max Mark	a	b	c	d	e	u
H867	01 Introduction to quantitative reasoning	Raw	72	55	47	39	31	23	0
H867	02 Statistical problem solving	Raw	60	40	34	28	23	18	0
		Overall	132	103	88	73	59	45	0

Advanced Free Standing Mathematics Qualification (FSMQ)

			Max Mark	a	b	c	d	e	u
6993	01 Additional Mathematics	Raw	100	59	51	44	37	30	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

			Max Mark	a	b	c	d	e	u
6989	01 Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0

Version	Details of change
1.1	Correction to Overall grade boundaries for H866
	Correction to Overall grade boundaries for H867